Using Confidence Intervals to Minimize Forecasting Error

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Motivation

- In past years there have been numerous requests from the Legislature for standard error analysis of corporation income tax.
Objectives

- Select economic variables that model past revenue well.
- Select economic variables that IHS consistently predicts well.
- Place some level of certainty on a revenue estimate.
- This will allow for comparison of models that use IHS economic forecasts as the main predictors.
Background:
FY 2014=$2,077 million

FY 2014 General Fund Revenue = $2,077.0

- Individual Income Tax 51%
- Property Tax 12%
- Remaining Sources 14%
- Oil & Natural Gas Taxes 5%
- Corporation Tax 7%
- Vehicle Taxes & Fees 5%
- Insurance Tax 3%
- Video Gaming Tax 3%
Volatility of Corporation Tax
Sources of Forecasting Error

- Taxpayer Behavior
  - Montana law allows corporations to carry back current year losses for three years, and carry forward losses for up to seven years.
- Reliance on a limited number of large taxpayers.
- Random Error
- Inherent error of IHS variables used for modeling.
Corporation tax liabilities are divided into numerous sectors.
These sectors include, but are not limited to, mining, manufacturing, retail trade, and financial sectors.
IHS economic variables are used to model each sector individually.
Sector estimates are combined to form a final revenue estimate for corporation tax liability.
Study Sector–by–Sector Error

\[ \epsilon \text{ (error)} = \text{actual value} - \text{estimate/actual value} \]
Example: Manufacturing Sector

[Graph showing CY Manufacturing Sector Corporation Tax Liability from 1995 to 2011]
WTI Price Fits Historical Data Well...

![Graph showing CY Standardized Manufacturing Tax Liability & WTI Price from 1995 to 2011. The graph includes two lines representing Tax Liability (black) and WTI (blue). The Tax Liability line shows a steady increase, while the WTI line fluctuates with peaks and troughs.](image)
...but how well do IHS forecasts of WTI predict manufacturing tax liability?
# Actuals vs. Estimates

<table>
<thead>
<tr>
<th>CY</th>
<th>First Year</th>
<th>Second Year</th>
<th>Third Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>50%</td>
<td>50%</td>
<td>N/A</td>
</tr>
<tr>
<td>2005</td>
<td>45%</td>
<td>67%</td>
<td>70%</td>
</tr>
<tr>
<td>2006</td>
<td>29%</td>
<td>57%</td>
<td>73%</td>
</tr>
<tr>
<td>2007</td>
<td>33%</td>
<td>48%</td>
<td>70%</td>
</tr>
<tr>
<td>2008</td>
<td>-5%</td>
<td>13%</td>
<td>49%</td>
</tr>
<tr>
<td>2009</td>
<td>1%</td>
<td>-44%</td>
<td>-40%</td>
</tr>
<tr>
<td>2010</td>
<td>-6%</td>
<td>3%</td>
<td>-17%</td>
</tr>
<tr>
<td>2011</td>
<td>4%</td>
<td>11%</td>
<td>3%</td>
</tr>
</tbody>
</table>
A confidence interval for the individual errors would require that their distribution be known.

With such a small sample, confidence in the true distribution is small.
Error Summary and Distribution

| Summary Statistics of Manufacturing Tax Liability Error Term by Estimate Year |
|-------------------------------|--------|--------|--------|
| Statistic                     | First Year | Second Year | Third Year |
| Mean                          | 19%    | 26%    | 30%    |
| Standard Deviation            | 23%    | 37%    | 47%    |

Histogram of Manufacturing First Year Estimates' Error Terms

Frequency

0 0.1 0.2 0.3 0.4 0.5 0.55

LFD
MONTANA LEGISLATIVE FISCAL DIVISION
Bootstrap Sample Mean

| Summary Statistics of Manufacturing Tax Liability Average Error Term by Estimate Year |
|---------------------------------|-----------------|-----------------|-----------------|
| Statistic                       | First Year      | Second Year     | Third Year      |
| Mean Error                      | 19%             | 26%             | 30%             |
| Standard Error                  | 8%              | 12%             | 16%             |

Histogram of First Year Estimates’ Mean Error Terms
Aggregate Results

<table>
<thead>
<tr>
<th>CY Corporation Income Tax Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>95% Confidence Intervals for the Aggregate Average Error Term of the Sector-Based Estimate</td>
</tr>
<tr>
<td>($ Millions)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>$135.0</td>
<td>$134.2</td>
<td>$165.9</td>
<td>24%</td>
</tr>
<tr>
<td>2016</td>
<td>$139.2</td>
<td>$129.6</td>
<td>$175.3</td>
<td>33%</td>
</tr>
<tr>
<td>2017</td>
<td>$145.3</td>
<td>$124.1</td>
<td>$186.9</td>
<td>43%</td>
</tr>
</tbody>
</table>
Fiscal Year Results

FY Corporation Income Tax Liability
Using 95% Confidence Intervals for the Sector-Based Estimate’s Average Error
($ Millions)

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>$149.7</td>
<td>$149.0</td>
<td>$177.7</td>
<td>19%</td>
</tr>
<tr>
<td>2016</td>
<td>$147.9</td>
<td>$147.2</td>
<td>$176.2</td>
<td>20%</td>
</tr>
<tr>
<td>2017</td>
<td>$151.7</td>
<td>$142.9</td>
<td>$184.8</td>
<td>28%</td>
</tr>
</tbody>
</table>
Are the Variables Independent?

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>% Range</th>
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</thead>
<tbody>
<tr>
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<td>29%</td>
</tr>
<tr>
<td>2016</td>
<td>$147.9</td>
<td>$140.0</td>
<td>$183.3</td>
<td>29%</td>
</tr>
<tr>
<td>2017</td>
<td>$151.7</td>
<td>$133.9</td>
<td>$193.9</td>
<td>40%</td>
</tr>
</tbody>
</table>

FY Corporation Income Tax Liability
95% Confidence Intervals for the Sector-Based Error Assuming Partial Dependence
($ Millions)
Results

- Adjusted corporation sector liability models to minimize future error bounds.
- Produced an aggregate estimate and corresponding error bounds for the sector-specific model.
- Calculated error bounds associated with alternative models.
- Allows for comparison of forecasts that use economic forecasts as the main predictors.
Sector-Based Model

FY Corporation Tax Liability with 95% Confidence Interval for Average Error

- Sector Model
- Upper Error Bound
- Lower Error Bound
Single Variable Model: WTI

FY Corporation Tax Liability Estimate with 95% Confidence Interval for the Average Error

- Single Variable (WTI) Model
- Upper Error Bound
- Lower Error Bound
Single Variable Model: Personal Income

FY Corporation Tax Liability Estimate with 95% Confidence Interval for the Average Error

- Personal Income Model
- Upper Error Bound
- Lower Error Bound
Single Variable Model: US Corporate Profits

FY Corporation Tax Liability with 95% Confidence Interval for the Average Error

- U.S. Corporate Profits Model
- Upper Error Bound
- Lower Error Bound
Lagged Corporate Profits Model

FY Corporation Tax Liability with 95% Confidence Interval for the Average Error

- Lagged U.S. Corporate Profits
- Upper Error Bound
- Lower Error Bound

$ Millions

2012 2013 2014 2015 2016 2017
Future Work

- Continue peer-review process.
- Incorporate relevant suggestions.
- Apply methodology to other large or volatile revenue sources.