Determining an Adequate Sample Size in Conjunction with Establishing Precision and Confidence

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Differences Between Attribute and Variable Sampling

- Attribute sampling is for estimating a proportion of a population that possesses a given attribute.
- Examples:
  - Proportion of red cars in Denver
  - Proportion of voters who support a particular resolution
  - Proportion of valid tax exemption certificates

Differences Between Attribute and Variable Sampling (Continued)

- Variable sampling is for estimating the parameters of a continuous variable.
- Examples:
  - Total tax due for an audit period
  - Ratio factor for a SUTCA (Taxable $/Total $)
- Attribute sampling is not appropriate for projecting tax liability in sales and use tax audits.
- Sample size calculations are different.

About Definitions

- It is critical that all terms be defined so that everyone knows and understands each term, how it is measured or estimated and how it is used.

Precision and Relative Precision

- In this talk:
  - Precision measures the sampling error in estimating a parameter. It is the product of the standard error of estimation and a value depending on the confidence level and the sampling distribution of the estimator.
  - Relative precision is the precision expressed as a percentage of the estimated value.
- Precision statements should always include the confidence level.
- Small improvements in precision or relative precision may require relatively large increases in sample size.

Confidence Level

- This refers to the (1-2α)100% two-sided confidence interval for the true value.
- For symmetric confidence intervals, the relative precision is half the width of the confidence interval divided by the estimated value, expressed as a percentage.

Error Rate

- In this talk, error rate, p, can be used two ways:
  - For estimating the error in tax paid or the taxable amount:
    - The proportion of items in the sampling frame that have errors in tax paid or taxable amount. Here, p is usually small.
  - For taxability:
    - The proportion of items in the sampling frame that are taxable. Here, p is usually moderate to large.
- This is usually unknown but may be estimated by:
  - A pilot sample
  - History of previous audits with this taxpayer
  - Experience of the CAS and/or Auditor

Considerations

The number of zeroes in the sample has a great impact on relative precision and the "error rate" parameter for the expected error rate model. That, in turn, depends on what the auditor is recording in the audit.

Recording	Number of Zero Values	Model "Error Rate"
Taxable Amount	Small to Moderate	Expected proportion of taxable items
Error in Reported Taxable Amount	Large	Expected proportion of errors
Error in Tax Paid	Large	Expected proportion of errors

Variability and Sample Size

- The more dispersed a population is, the larger the sample must be to reach a given precision and confidence level.
- Question: How many sample items must be taken to estimate (within 10% at 95% confidence) the mean age of
  - The residents of Denver?
  - The first grade students of Denver public schools?

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Allocating Sample Resources

- Assuming the cost of sampling is the same from item to item, then strata with greater variability will get more of the sample than those with smaller variability.
- In sales and use tax audits, it is assumed the cost of auditing items is constant across the sampling frame.
- The formula for allocating resources in the most efficient way is known as the Neyman allocation.

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Precision, Confidence Level, Error Rate and Sample Size

- Improving precision or increasing confidence level increases sample size for a fixed error rate.
- When relative precision is defined as in this talk, smaller error rates require larger samples for a fixed relative precision and confidence level.
- Serious under sampling can occur if the error rate is ignored.

Expected Error Rate Model

- Assumptions:
  - Errors are random and independent of the item being audited.
  - The error rate is constant over the entire sampling frame. (Note: Theoretically, a different rate could be applied to each stratum. I don't think it is worth the effort.)
  - All the errors are in the same direction.
- The last assumption is critical
  - For estimating taxability it is usually no problem.
  - For error in tax paid it may not be true. (We'll discuss this further later on.)

Expected Error Rate Model (Continued)

- This model adjusts the mean and standard deviations which are used in the formula for sample size.
- Model: W = XY where
  - X is invoice amount
  - Y is Bernoulli random variable with $P(Y=1) = p$ and $P(Y=0) = 1-p$, $0 < p < 1$
- A detailed explanation of the model is included in the workshop package.

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When Should I Use the Model?

- $Var(W) > Var(X)$ when $p > (cv)^2$ so one would think only when this is the case.
- It is not so simple as looking at variances since the model also models the total of the errors or total taxable amount.
- When precision in estimating the total is also taken into account, the model can make a big difference even when $p < (cv)^2$.

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Heuristic Example of EER Impact

- Goal: Estimate taxable amount to within 10% with confidence 95%
- Total of amounts in sampling frame: \$100M
- 20% are taxable, 80% are not
- Goal
  - Without Model: ± \$10M
  - With Model: ± \$2M
- A detailed example of EER Model impact along with an alternative definition of precision and application of the model is in the handout.

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Putting It Together

- Define the stratification and calculate stratum statistics.
- Determine which strata are to be 100% detailed.
- Apply the Expected Error Rate model
  - If you are not sure whether or not to apply the model then run the sample size calculations both with and without the expected error rate. Use the larger value.

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Putting It Together (Continued)

- Calculate required sample size based on desired precision (or relative precision), confidence level and expected error rate.
- Use the Neyman allocation formula to optimally allocate the sample resources to the strata.
  - Remove 100% detail strata and reduce sample size appropriately.
  - Apply Neyman formula
  - Any stratum allocated more sample than actual items is changed to 100% detail and the process is repeated, including recalculation of the sample size.

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Putting It Together (Continued)

  - Apply minimum sample size requirements
- What if the sample exceeds available audit resources? Start over.
- The CAS must balance stratification, precision and confidence level to get a sample size that is acceptable.
- Always use the most precise (or most conservative and reasonable) estimate of error rate.

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Minimum Sample Sizes

- Not usually a problem when estimating taxability (because of fewer zero values).
- Depends on policy about minimum number of errors to project a tax due or refund.
- Expected Number of Errors in a stratum = Expected Error Rate × Stratum Sample Size
- Example: EER = 1%, Sample Size = 100, Expected Number of Errors = 1

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Probability of At Least 3 Errors

Sample Size	Error Rate				
	1%	2%	3%	4%	5%
30	.003	.022	.060	.117	.187
50	.014	.078	.189	.323	.459
75	.040	.190	.392	.581	.730
100	.079	.323	.520	.768	.882
200	.323	.765	.941	.988	.998

Note: Calculations are based on binomial distribution.

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More Considerations

- Total sample size can be very large if there are numerous strata and a moderately large minimum sample.
- Often, more strata lead to lower sample sizes – until minimum sample sizes are applied.
- Depending on precision and confidence level, you may have every non-detailed stratum at the minimum sample size.

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Error Structure and Relative Precision

If the errors

- occur randomly with probability p, independent of the item being sampled;
- are randomly positive or negative with probability π, independent of the item being sampled;

then

- relative precision can be very poor;
- increasing sample size may not help.

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