Model of Sales & Use Tax File

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The models presented below can be used in helping to determine a sample size. The models represent a realistic approximation to actual use tax and sales tax audits.

Specifically, it is assumed that a certain percentage of the taxpayer's records are incorrect-either taxable when the taxpayer has said it was tax exempt or nontaxable when the taxpayer has said it was taxable. Because it is not known which records are in error, the model assumes that all records are equally likely to be in error. In Model I all the errors are of one kind or the other, while both types of errors can occur in Model II. The frame being sampled can then be regarded as one of the possible outcomes of the model. This is similar to regarding the result of twenty tosses of a fair coin as being one of the possible outcomes of a binomial model with n=20 and p=1/2.

The models enable us to calculate a mean error amount and the variance of the error amounts. Of course, the actual average and variance of error amounts in the frame can differ from these calculated values. In similar fashion, the number of heads and tails observed in 20 tosses of a fair coin can differ from the number expected using the binomial model (10 heads and 10 tails). For this reason, using the model for determining sample size can be useful but the results are only good approximations.

Model I: Errors Are All Positive or All Negative

Assumptions: X = invoice file amount with $E(X) = \mu$, $Var(X) = \sigma^2$ and Y is Bernoulli random variable with $P(Y = 1) = p$ (error rate). X and Y are jointly independent.

This model assumes that the error amount W either equals the taxable amount when Y was incorrectly classified as nontaxable (the nontaxable amount if X was regarded as incorrectly taxable) or the error in the amount of tax. In either case, Y = 1 when an error occurs.

Define W = XY. Then $E(W) = \mu_W = \mu p$ and $\sigma^2_W = p\sigma^2 + p(1 - p)\mu^2$.

Note: The variance for W is larger than the variance for X whenever $p \geq \frac{\sigma^2}{\mu^2} = cv^2$.

If p = 0, there are no errors and the auditor will find none. If p = 1, then all the invoices are in error. If $(cv)^2 < 1$ then the variance is maximized when $p = \frac{\sigma^2 + \mu^2}{2\mu^2} = \frac{cv^2 + 1}{2}$.

If $cv^2 \geq 1$ then the variance for W is an increasing function of p and attains its maximum value when p = 1. In this case the variance of W can be less than or equal to the variance of X.

The model may be used to determine a sample size that nearly achieves a specified goal. This goal may be selected to represent a desired precision (the difference between a lower confidence limit and a point estimate) or as a desired relative precision (precision divided by the expected total error). If the desired relative precision is specified, the desired relative precision is multiplied by the expected error to obtain the desired precision. When this is done, the results apply whether the auditor is estimating the error in the taxable amount or the error in the tax as long as the tax rate is constant for all units in the frame.

Another way of determining the desired precision is to set the desired precision equal to specified percentage of the taxpayer's recorded taxable amount. Doing this the total estimated taxable amount would have a margin of error equal to the specified percentage. This could also be done when the auditor elects to estimate the error in tax by specifying an allowable percentage of the tax paid by the taxpayer.

The resulting sample sizes will be different as the following example illustrates. In this example, the file contains 19,912 sampling units divided among six strata. The total recorded amount is $19,736,162.The taxpayer has classified about 40% of this amount as taxable ($7,894,465). The auditor's objective is to estimate the error in the taxable amount. All errors represent amounts not regarded as taxable which should have been. The example can also be used when the auditor's objective is to estimate the error in the amount of tax.

To illustrate the effects of specifying the relative precision, the auditor specifies that the desired precision should be 5% of the expected error. For the alternative, the desired precision should be 1% of the taxpayer's taxable amount.

For both, the top stratum of 63 items is to be sampled 100% and the confidence level used is 95% (one-sided).

In the example, only the initial total sample sizes will be compared. Thus, the prospect that allocating the total sample size to the strata results in additional strata being sampled 100% is ignored.

Table A1: Example Stratification Diagnostics

Stratum	Range ($)	Size	Mean	Std. Dev.	CV	$(CV)^2$
1	10.00-99.99	9683	45.78	24.36	.5320	.2830
2	100.00-999.99	8,009	311.12	215.08	.6913	.4779
3	1,000.00-9,999.99	1,819	3,026.68	2,187.11	.7226	.5222
4	10,000.00-24,999.99	250	15,676.96	4,392.35	.2802	.0785
5	25,000.00-49,999.99	88	34,182.82	7,104.32	.2078	.0432
6	50,000.00-96,880.00	63	69,337.39	13,793.13	.1989	.0396

The following table shows the required sample sizes for the specifying the desired relative precision using six different values for the expected error percentage. In each case, the sample size reflects auditing the top stratum 100%. The model results in an expected error that depends on the expected error percentage. Thus, for each expected error rate, the expected error amount equals the expected error percentage times $19,736,162. For example, for an expected error rate of 20%, the expected error amount is $3,947,232. In this case the desired precision at 95% one sided confidence is $197,362 because the desired precision is set at 5% of $3,947,232. As the expected error percentage decreases the desired precision decreases. Thus at 1%, the desired precision is $9,868.

EXPECTED ERROR PERCENTAGE	20%	5%	4%	3%	2%	1%
SAMPLE SIZE	1156	1466	1490	1514	1538	1563

To illustrate the alternative, the same six expected error rates are used, but the desired precision depends not on the expected error amount but on the amount the taxpayer has regarded as taxable. Because the taxpayer considers the taxable amount to be $7,894,465, the auditor sets the desired precision at $78,945 which presents the desired 1%. This desired precision amount remains the same regardless of the expected error percentage. Because in the model, the expected variance of errors decreases as the percentage of errors decreases, the sample sizes decrease as the percentage of errors decrease. This is reflected in the following table showing the sample sizes.

EXPECTED ERROR PERCENTAGE	20%	5%	4%	3%	2%	1%
SAMPLE SIZE	1532	1300	1244	1161	1027	770

Model II: Errors Can Be Positive or Negative

Assumptions: X = invoice file amount with $E(X) = \mu$, $Var(X) = \sigma^2$, Y is Bernoulli random variable with $P(Y = 1) = p$ (error rate), and Z is a discrete random variable (indicating the direction of error) with values {-1, 1} with $P(Z = 1) = \pi$, $P(Z = -1) = 1 - \pi$. The random variables X, Y and Z are jointly independent. In this model error amounts with a positive sign signify a transaction amount incorrectly treated as nontaxable and an error amount with a negative sign represents a transaction amount treated as taxable that was nontaxable.
Define W = XYZ. Then $E(W) = \mu_W = \mu p(2\pi-1)$ and $\sigma^2_W = p\sigma^2 + p(1 - p)\mu^2 + 4p^2 \pi(1 - \pi) \mu^2$.

If $\pi = ½$, $E(W) = 0$ and $\sigma^2_W = p(\sigma^2 + \mu^2)$. The limiting value of σ^2_W as π goes to either 0 or 1 is $\sigma^2_W = p\sigma^2 + p(1 - p)\mu^2$.

It is interesting to look at the coefficient of variation for W:

$$cv = \frac{\sqrt{p\sigma^2 + p(1-p)\mu^2 + 4p^2\pi(1-\pi)\mu^2}}{p(2\pi - 1)\mu}$$

Then, the coefficient of variation for W becomes unbounded as $\pi \rightarrow ½$. This means that when the frequency of the two types of misclassification is nearly equal, no sample size can be specified that achieves a desired relative precision regardless of the error percentage.

Otherwise, this model can be used to determine sample sizes in a manner similar to using Model I.