

Options to Allocate Results of a Statistical Audit to Local Tax Jurisdictions or Different Department Budgets

Dr. Richard W. Kulp

This material is the property of the presenter and cannot be reproduced or used without the expressed written consent of the presenter.

VIII-1

Estimating Totals Over Subpopulations (Domains)

When the following is met:

1. The subpopulations are represented in all strata;
2. Each subpopulation has a sample value in each stratum;
3. The number of items in each subpopulation is known for each stratum,

there is a well-known solution (see section 5A.14 in Cochran's **Sampling Techniques**, 3rd Ed.

Note: This may not solve the local tax allocation problem for complex tax situations.

VIII-2

When All Conditions Are Met

$$\widehat{Y}_j' = \sum_h N_{hj} \bar{y}_{hj} = \sum_h N_{hj} \sum_{i=1}^{n_{hj}} \frac{y_{hij}}{n_{hj}}$$

Where:

Y_j is the total for domain j

N_{hj} is the number of items in domain j within stratum h

n_{hj} is the number of sample items in domain j within stratum h

y_{hij} is the i^{th} sample item in domain j within stratum h

VIII-3

When All Conditions Are Met (continued)

- The variance of the estimated domain total is

$$V(\widehat{Y}_j') = \sum_h \frac{N_{hj}^2 S_{hj}^2}{n_{hj}} \left(1 - \frac{n_{hj}}{N_{hj}}\right)$$

VIII-4

When the N_{hj} Are Unknown

$$\widehat{Y}_j = \sum_h \frac{N_h}{n_h} \sum_{i=1}^{n_{hj}} y_{hij}$$

$$v(\widehat{Y}_j) = \sum_h \frac{N_h^2}{n_h(n_h - 1)} (1 - f_h) \left[\sum_{i=1}^{n_{hj}} y_{hij}^2 - \frac{(\sum y_{hij})^2}{n_h} \right]$$

Note: The term in square brackets is not the usual variance.

VIII-5

Example Taxable Amount

Strat	Lower	Upper	Size	D1 Size	D2 Size	Sample Sizes		
						D1	D2	Total
1	10.01	100.00	500	200	300	8	12	20
2	100.01	1000.00	300	100	200	4	6	10
3	1000.01	10000.00	200	75	125	12	8	20
Totals			1000	375	675	24	26	50

VIII-6

Example Data (Non-zero Values)

Stratum	N_h	N_h	Domain 1	Domain 2
1	500	20 (6 non-zero, 14 zero)	10 15 20	8 52 70
2	300	10 (5 non-zero, 5 zero)	150 322 900	100 875
3	200	20 (4 non-zero, 16 zero)	3210 8722 9818	1500
Totals	1000	50 (15 non-zero, 35 zero)		

VIII-7

Example Calculations—Stratum 1

	Domain 1		Domain 2	
	y_{hij}	y_{hij}^2	y_{hij}	y_{hij}^2
Non-zero Data	10 15 20	100 225 400	8 52 70	64 2704 4900
Sum	45	725	130	7668
$\frac{N_h}{n_h} \sum$	$25 \times 45 =$ 1125	393947.3684	$25 \times 130 =$ 3250	4309263.1579

$$\frac{N_h^2}{n_h(n_h - 1)} (1 - f_h) \left[\sum_i y_{hij}^2 - \frac{(\sum_i y_{hij})^2}{n_h} \right] = \frac{250000}{39800} \times \left(1 - \frac{20}{500}\right) \times \left[725 - \frac{45^2}{20}\right] = 393947.3684$$

$$\frac{N_h^2}{n_h(n_h - 1)} (1 - f_h) \left[\sum_i y_{hij}^2 - \frac{(\sum_i y_{hij})^2}{n_h} \right] = \frac{2500}{39800} \times \left(1 - \frac{20}{500}\right) \times \left[7668 - \frac{130^2}{20}\right] = 4309263.1579$$

VIII-8

Example Calculations Concluded

Stratum	Domain 1		Domain 2	
	Total	Variance	Total	Variance
1	1125	393947.3684	3250	4309263.1579
2	41160	813995973.3333	29250	429828947.3684
3	217500	15074278389.4737	15000	202500000.0000
Total	259785	1588668310.1754	47500	636638210.5263
Standard Deviation		126050.2610		25231.6906

VIII-9

Other "Less Statistical" Ways

- Use proportions based on some "Base"
 - Tax Return History
 - Sampling Frame
 - Pull List

VIII-10

Decide Which Base to Use

- Tax Return History
 - Most "removed" from audit
 - Requires detail for each locality
 - Allocation does not consider where errors occurred
- Sampling Frame
 - Somewhat "removed" from audit
 - Required detail for each locality must be in records to be sampled
 - Allocation does not consider where errors occurred.

VIII-11

Decide Which Base to Use (Continued)

- Pull List (Sample)
 - Uses actual audit results
 - Allocation is dependent on where errors occurred.

VIII-12

Local Jurisdictions (Once Base is Determined)

- When all local jurisdictions have the same tax rate:
 - For each local jurisdiction
 - From the base calculate the ratio of taxable amounts to total taxable
 - Apply that ratio to the assessment
 - Example: A jurisdiction has 25% of the dollars in the base. They would get 25% of the local tax assessment.

VIII-13

Local Jurisdictions (Continued)

- When the tax rate differs from jurisdiction to jurisdiction:
 - Auditor should separate out local tax at each rate.
 - Apportion the local tax assessment by the ratio of taxable dollars/total dollars at each tax rate.
 - For a given tax rate, apportion that tax rate's proportion of local tax among all jurisdictions with that tax rate, as before.

VIII-14

Departmental Budgets

- Requires Departmental Level Detail
 - Base is sampling frame or pull list
- For each department/budget
 - From the base calculate the ratio of taxable amounts to total taxable
 - Apply that ratio to the assessment

VIII-15