

What Happened To The Precision?

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Confidence Level

- This refers to the $(1-2\alpha)100\%$ two-sided confidence interval for the true value.
- For symmetric confidence intervals, the relative precision is half the width of the confidence interval divided by the estimated value, expressed as a percentage.

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Precision vs. Relative Precision

- Precision is a value calculated from the standard error of the estimate and a number based on the confidence level and probability distribution of the estimator.
- Relative Precision is the ratio of precision to the total.
- Precision and Relative Precision are estimated by substituting sample estimates for the theoretical values.

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What Decreases Precision?

- Increasing standard error (standard deviation of estimator) by itself makes both precision and relative precision worse.
- Decreasing the absolute value of estimated tax due by itself
 - makes relative precision worse
 - does not affect precision.

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What Increases Standard Error?

- Including positive and negative invoices sometimes increases standard error of estimation.
- Under sampling (N is too small) since standard error of estimation is inversely proportional to the square root of sample size.

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What Decreases Total Tax Due?

- Fewer errors than expected.
- Errors that are randomly positive and negative so the net is “close” to zero.

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Error Rate

- In this talk, error rate, p , is
 - The proportion of items in the sampling frame that have errors in tax paid. Here, p is usually small.
- This is usually unknown but may be estimated by:
 - A pilot sample
 - History of previous audits with this taxpayer
 - Experience of the CAS and/or Auditor

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Known vs. Unknown

- Known:
 - File of invoice amounts (Sampling Frame)
- Unknown:
 - File of errors in tax paid.
- Sample size is calculated on the known information and assumptions about the relationship between the known and the unknown.

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Expected Error Rate Model I

- Assumptions:
 - Errors are random and independent of the item being audited.
 - The error rate is constant over the entire sampling frame. (Note: Theoretically, a different rate could be applied to each stratum. I don't think it is worth the effort.)
 - All the errors are in the same direction.
- The last assumption is critical
 - For estimating taxability it is usually no problem.
 - For error in tax paid it may not be true.

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Expected Error Rate Model I (Continued)

- This model adjusts the mean and standard deviations which are used in the formula for sample size.
- Model: $W = XY$ where
 - X is invoice amount
 - Y is Bernoulli random variable with $P(Y=1) = p$ and $P(Y=0) = 1-p$, $0 < p < 1$
- A detailed explanation of the model is included in the workshop package.

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Expected Error Rate Model II

- All of model I plus an independent Bernoulli random variable indicating the direction of the error, positive or negative.
- Model: $W = XYZ$ where
 - X is invoice amount
 - Y is Bernoulli random variable with $P(Y=1) = p$ and $P(Y=0) = 1-p$, $0 < p < 1$
 - Z is Bernoulli random variable with $P(Z=1) = \pi$ and $P(Z=0) = 1-\pi$

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Implications of Model II

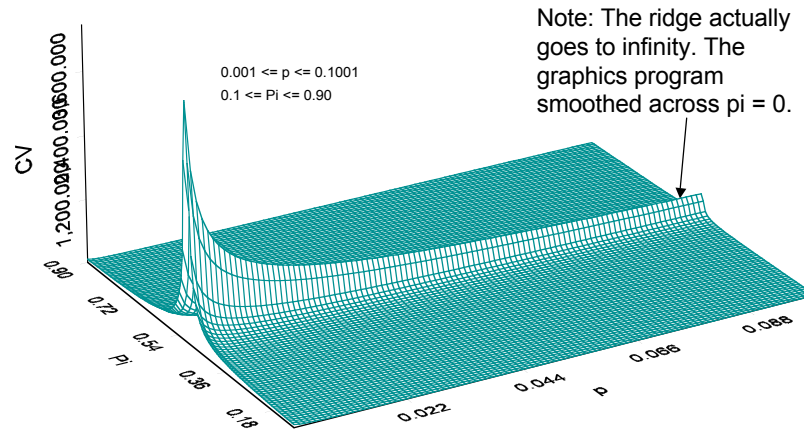
- When the errors are randomly positive and negative with $\pi = \frac{1}{2}$ the theoretical average error is always zero, regardless of the average invoice amount, μ , or the error rate, p , for $0 < p \leq 1$.
- The coefficient of variation (cv) is the ratio of the standard deviation to the mean. When $\pi = \frac{1}{2}$ it is unbounded.

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Expected Error Rate Model II

Coefficient of Variation for ABC Aircraft Fabricators Data



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Practical Implications

- When the errors are all (or mostly) the same sign, increasing sample size can improve both precision and relative precision.
- When the errors are randomly positive and negative so that the net is “close” to zero, increasing sample size will not improve relative precision, but should improve precision.

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